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Immanuel College

Year 12 AS Maths Induction Booklet

Induction Booklet Instructions

The transition for GCSE to A-Level is a considerable jump. To ensure the best start at A Level we expect all students to complete the following activities focusing on some of the key fundamentals that your course will be built upon.

On the following pages you will find several key skills with examples given then activities to complete. Please can you ensure that all these activities are completed with your workings shown.

Please bring this booklet with you to your first Maths lesson in September for your teacher to check over.

If you require any further help or find there are some skills that you are struggling with, then Hegarty Maths also has a transition course that you can complete. This course has a range of videos that will help you to develop these skills to ensure the best possible start in September.

This course can be found via the Hegarty Maths YouTube page, the playlist is labelled as 'Live Lessons'.

https://youtube.com/playlist?list=PLxHVbxhSvleR5tntP2FxYBJCoY5-pD_Z8

A-Level Preparation **Laws of Indices**

The laws of indices are shortcuts for simplifying expressions involving indices without evaluating them.

Law 1: $a^x \times a^y = a^{x+y}$

E.g. Simplify $3^5 \times 3^2$. Give your answer in index notation.

Firstly, remember what an index is – an instruction to multiply a number by itself.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$

$3^2 = 3 \times 3$

$3^5 \times 3^2 = \underline{3 \times 3 \times 3 \times 3 \times 3} \times \underline{3 \times 3} = 3^7$

The first index law lets you skip the intermediate steps and go straight to $3^5 \times 3^2 = 3^7$

It's important to remember that all three of these laws work with both numerical and algebraic bases. Read the question carefully. If it says evaluate, you are expected to give a numerical answer without an index. If it says index notation, you are expected to give it in the form a^b.

E.g.: $x^3 \times x^5 = x^8$

$5^a \times 5^{2b} = 5^{a+2b}$

You can only use these laws if both bases are the same:

E.g.: $3^5 \times 5^2 \neq 15^7$

$a^5 \times b^3 \times a^2 \times b^3 = a^7 \times b^6$

Law 2: $a^x \div a^y = a^{x-y}$

E.g. $a^7 \div a^4$

$$= \frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a}$$

$$= \frac{a \times a \times a \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}$$ (cancel out where possible)

$= a^3$

Or, using the index law as a shortcut: $a^7 \div a^4 = a^{7-4} = a^3$

Again, remember that this will work with both numerical and algebraic terms. Be very careful with negative numbers. It is easy to make mistakes when dealing with negative indices.

E.g.: $5^4 \div 5^{-3} = 5^{4--3} = 5^{4+3} = 5^7$

$x^{-5} \div x^7 = x^{-5-7} = x^{-12}$

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Law 3: $(a^x)^y = a^{x \times y}$

E.g. $(5^3)^4$

$= \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

$= 5^{12}$

Or, using index law 3: $(5^3)^4 = 5^{3 \times 4} = 5^{12}$

Again, be careful of negative numbers. If you have to apply more than one of these laws, in most cases, it is easier to apply them in order.

E.g.: $\left(\frac{(a^4 \times a^3)}{(a^2)}\right)^2$

$= \left(\frac{(a^7)}{(a^2)}\right)^2$ Use the first law on the numerator.

$= (a^5)^2$ Then use the second law to remove the fraction.

$= a^{10}$ Then use the third law to get a final answer.

Here are some to try:

1. Simplify each expression. Give your answers in index form.

   a. $5^4 \times 5^8$ b. $m^4 \div m^2$ c. $(a^3)^2$ d. $3^5 \times 3$

2. Simplify each expression. Give your answers in index form.

   a. $3^8 \times 3^{-2}$ b. $\frac{h^{-3}}{h^5}$ c. $p^{-2} \div p^{-9}$ d. $(5^{-3})^{-2}$

3. Simplify each expression. Give your answers in index form.

   a. $3a^2 \times 3a^5$ b. $(3x^4)^3$ c. $\frac{12x^3}{4x^5}$ d. $a^2b^5 \times a^4b^{-8}$

4. Simplify the expression. Give your answer in index form.

   $\left(\frac{3a^5 \times 6a^{-7}}{2a^5}\right)^2$

BEYOND MATHS

A-Level Preparation **Evaluating Negative and Zero Powers**

An index is an instruction telling you how many times to multiply a number by itself, e.g. $5^3 = 5 \times 5 \times 5$. However, that index doesn't have to be a positive whole number; it can also be negative, zero or even a fraction. Look at the pattern of these indices to see how this works:

$$\begin{array}{ccccc}
 & 2^3 & = & 2 \times 2 \times 2 & \\
\times 2 \uparrow & & & & \downarrow \div 2 \\
 & 2^2 & = & 2 \times 2 & \\
\times 2 \uparrow & & & & \downarrow \div 2 \\
 & 2^1 & = & 2 & \\
\times 2 \uparrow & & & & \downarrow \div 2 \\
 & 2^0 & = & 1 & \\
\times 2 \uparrow & & & & \downarrow \div 2 \\
 & 2^{-1} & = & \frac{1}{2} & \\
\times 2 \uparrow & & & & \downarrow \div 2 \\
 & 2^{-2} & = & \frac{1}{2 \times 2} & \\
\times 2 \uparrow & & & & \downarrow \div 2 \\
 & 2^{-3} & = & \frac{1}{2 \times 2 \times 2} &
\end{array}$$

The first thing to notice is that $2^0 = 1$. Anything with an index of 0 is 1, whether the base is large, small or algebraic.

Example 1

$0.5^0 = 1$

$19\,251\,215^0 = 1$

$(3x^2 + 8x + 10)^0 = 1$

Secondly, when we are dealing with negative indices, we are effectively dividing instead of multiplying.

$2^3 = 8$

and $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

This means that a negative power tells us to find the reciprocal of the number. Remember, the reciprocal of a number is what we get when we divide 1 by that number:

The reciprocal of x is $\frac{1}{x}$.

If the number is already a fraction, this is the same as inverting the numerator and denominator:

The reciprocal of $\frac{x}{y}$ is $\frac{y}{x}$.

Example 2

Evaluate 4^{-3}

The negative tells us to find the reciprocal. The reciprocal of 4 is $\frac{1}{4}$.

$4^{-3} = (\frac{1}{4})^3 = \frac{1}{4^3} = \frac{1}{64}$

Notice that we are cubing both parts of the fraction, but since 1^3 is 1, we just write 1.

Likewise, if the base is a fraction, we find the reciprocal and then apply the index to both the numerator and denominator.

Example 3

a. Evaluate $(\frac{3}{5})^{-2}$

The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

$(\frac{3}{5})^{-2} = \frac{5^2}{3^2} = \frac{25}{9}$

b. Evaluate $(\frac{1}{2})^{-2}$

$(\frac{1}{2})^{-2} = \frac{2^2}{1^2} = \frac{4}{1} = 4$

c. Evaluate $(\frac{2}{5})^{-3}$

$(\frac{2}{5})^{-3} = \frac{5^3}{2^3} = \frac{125}{8}$

Your Turn:

1. a. What is $3^5 \div 3^2$ in index form?

b. What is $3^2 \div 3^2$ in index form?

c. Evaluate $3^2 \div 3^2$

d. Evaluate 3^0

e. Evaluate 27.54^0

f. Evaluate $2.7523^0 \times 268^1 \times 892^0$

2. Evaluate the following:

a. 5^{-2}

b. 8^{-2}

c. 3^{-3}

d. 2^{-5}

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3. Write each in index form:

a. $\frac{1}{16}$ b. $\frac{1}{49}$ c. $\frac{1}{125}$ d. $\frac{1}{1000}$

4. Evaluate, giving your answers as fractions in their simplest form:

a. $(\frac{3}{5})^{-1}$ b. $(\frac{7}{8})^{-2}$ c. $(\frac{1}{4})^{-3}$ d. $(\frac{2}{3})^{-3}$

5. Evaluate, giving your answers as fractions in their simplest form:

a. $(3x)^{-2}$ b. $(2x^3)^{-2}$ c. $(5x^4)^{-3}$ d. $(2x^2y^3)^{-4}$

A-Level Preparation **Evaluating Fractional Powers**

Fractional powers (fractional indices) are a way of representing a combination of roots or powers. The denominator represents the value of the root.

For example, a fractional power of $\frac{1}{2}$ represents a square root and $\frac{1}{3}$ represents a cube root.

Example 1

$25^{\frac{1}{2}} = \sqrt{25} = 5$

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$(4x^2)^{\frac{1}{2}} = \sqrt{4x^2}$

$= \sqrt{4}\sqrt{x^2}$

$= 2x$

The numerator of a fractional power represents an integer power, which allows a fractional power to represent both a power and a root.

For example, a fractional power of $\frac{3}{2}$ would tell you to raise the base to a power of 3, and to square root the answer. These two operations can be done in either order; you can use your judgement to decide which is easier.

Example 2

$4^{\frac{3}{2}}$

$= \sqrt{4^3}$

$= \sqrt{64}$

$= 8$

or $4^{\frac{3}{2}}$

$= \sqrt{4}^3$

$= 2^3$

$= 8$

As you can see, the answer is the same whether you evaluate the power or the root first.

Example 3

$9^{\frac{3}{2}}$

$= \sqrt{9^3}$

$= \sqrt{729}$

$= 27$

When this is combined with negative powers, you are likely to have three operations to carry out. Again, the order will not change the result:

E.g. $(\frac{9}{25})^{-\frac{3}{2}}$

$= (\frac{25}{9})^{\frac{3}{2}}$

$= (\frac{\sqrt{25}}{\sqrt{9}})^3$

$= \frac{5^3}{3^3}$

$= \frac{125}{9}$

Your Turn:

1. Evaluate the following:

a. $36^{\frac{1}{2}}$ __________

b. $1000^{\frac{1}{3}}$ __________

c. $64^{\frac{1}{3}}$ __________

d. $81^{-\frac{1}{2}}$ __________

2. Evaluate the following:

a. $27^{\frac{2}{3}}$

b. $8^{\frac{4}{3}}$

c. $49^{\frac{3}{2}}$

d. $64^{\frac{2}{3}}$

3. Express in the form $a^{\frac{m}{n}}$, where m and n are integers.

a. $\sqrt{a^3}$

b. $\sqrt[3]{a^5}$

c. $\frac{1}{\sqrt{a^7}}$

d. $\sqrt{a} \times \frac{1}{\sqrt{a^5}}$

BEYOND MATHS

4. Write the following expressions in order, from smallest to largest:

$25^{\frac{1}{2}}$, $8^{\frac{2}{3}}$, $27^{\frac{1}{3}}$, $(\frac{1}{9})^{-\frac{3}{2}}$, $(\frac{1}{12})^{-1}$, $(27^{\frac{5}{3}})^{0}$,

5. Write $64^{\frac{2}{3}} \times 2^{3}$ in the form 2^{a}, where a is a positive integer.

BEYOND MATHS

A Level Preparation **Polynomials**

Polynomials appear throughout the AS and A level course. A polynomial is just an expression consisting of variables (like x or y), numbers and positive integer (whole number) powers. The terms in this expression can be added or subtracted. For example, $3x^2 + 2x - 4$ is a polynomial but $3x^{-4}$ is not.

Expanding (Multiplying) Brackets

You will need to be able to multiply out single, double and triple brackets.

To expand single brackets, multiply everything inside the bracket by the term on the outside.

E.g. Expand $3(2x + 5y)$

$3(2x + 5y) = (3 \times 2x) + (3 \times 5y)$

$\qquad = 6x + 15y$

To expand double brackets, multiply everything in the first bracket by everything in the second. The **FOIL** (**f**irst, **o**uter, **i**nner, **l**ast) method can be helpful in ensuring you don't miss any terms.

E.g. Expand $(2x - 7)(x - 1)$

F L

$$(2x - 7)(x - 1)$$

I

O

$(2x - 7)(x - 1) = (2x \times x) + (2x \times -1) + (-7 \times x) + (-7 \times -1)$

$\qquad = 2x^2 - 2x - 7x + 7$

$\qquad = 2x^2 - 9x + 7$

To expand triple brackets, begin by multiplying one pair of brackets, then multiply each term in the expanded expression by each term in the remaining bracket. You will need to be systematic in your approach so you don't lose any terms. A grid method can help.

E.g. Expand $(x - 1)(x + 2)(x + 3)$

$(x - 1)(x + 2)(x + 3) = (x^2 + 2x - x - 2)(x + 3)$

$\qquad = (x^2 + x - 2)(x + 3)$

	x^2	x	-2
x	x^3	x^2	$-2x$
3	$3x^2$	$3x$	-6

$(x^2 + x - 2)(x + 3) = x^3 + 3x^2 + x^2 + 3x - 2x - 6$

$(x - 1)(x + 2)(x + 3) = x^3 + 4x^2 + x - 6$

Notice that the constant term in the expansion is the product (that means times) of the numerical part in the brackets. $1 \times -2 \times 3 = -6$. This is important for helping us to factorise cubic expressions later on, but can be a nice way to check whether your work is likely to be correct.

Your Turn

Expand and simplify:

1. $5(2x - 7)$

2. $8x(2x + 3)$

3. $7a(3a + 2b - 4)$

4. $5(2x + 1) + 3(x + 4)$

5. $8y(y - 4) - 2y(3 - y)$

6. $(3x + 2)(x + 5)$

7. $(x - 4)(3x - 9)$

8. $(a + b)(b - c)$

9. $(3x + 2)^2$

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10. $(x + 8)(2x + y - 4)$

11. $(x + 3)(x + 4)(x + 1)$

12. $(2x - 5)(x - 2)(x + 7)$

13. $(x + 1)^3$

14. $(x + 2)^2(x + 5)$

Factorising

To *factorise* means to put an expression back into brackets. To do this, begin by taking out any common factors among the terms (that's where the word comes from!). Then divide each term by this number, or expression, to find the terms that go inside the brackets.

E.g. Factorise $8x^2 - 10x$

The common factor is $2x$, so $2x$ goes on the outside of the brackets. $8x^2 \div 2x = 4x$ and $10x \div 2x = 5$.

$8x^2 - 10x = 2x(4x - 5)$

A common misconception is to think that, because an expression contains more than two terms or has a squared term in it, it must factorise into double brackets. This is not always true.

E.g. Factorise $20p^2q + 5pq^2 - 15pq$

The common factor is $5pq$, so $5pq$ goes on the outside of the brackets.
$20p^2q \div 5pq = 4p$, $5pq^2 \div 5pq = q$ and $-15pq \div 5pq = -3$.

$20p^2q + 5pq^2 - 15pq = 5pq(4p + q - 3)$

You might even see an expression where the common factor is itself an expression.

E.g. Factorise $4(x + y) - p(x + y)$

The common factor is $(x + y)$. $4(x + y) \div (x + y) = 4$ and $- p(x + y) \div (x + y) = -p$.

$4(x + y) - p(x + y) = (x + y)(4 - p)$

Your Turn

Factorise fully:

1. $12x + 15$
2. $27x - 18$
3. $10y^2 + 28y$
4. $14ab + 21a$
5. $32x + 40y - 24$
6. $10x^2y - 15xy^2$
7. $12a^3b^2 + 18a^2b^3 - 27ab^4$
8. $a(b + c) + 5(b + c)$
9. $x(y + 3) + 2(y + 3)$
10. $2r(a - 4) - p(a - 4)$

A Level Preparation **Quadratic Expressions**

Quadratic expressions are of the form $ax^2 + bx + c$, where $a \neq 0$. a and b are called the coefficients of x^2 and x respectively. A quadratic is a polynomial, but since these appear so often in the AS and A-level course, they get their own section!

Factorising: When a = 1

When $a = 1$, the expression is $x^2 + bx + c$. If it can be factorised, this sort of expression will go into two brackets, with an x at the front of each. To find the numerical part, find two numbers that multiply to give c and add to give b.

E.g. Factorise $x^2 + 2x - 15$

Find two numbers that multiply to give -15 and add to give 2. List the factors of 15 then deal with the signs. The factors of 15 are 1 and 15, or 3 and 5.

A negative multiplied by a positive is a negative so one number in each factor pair will have to be negative. To give a sum of positive 2, we choose -3 and 5.

$x^2 + 2x - 15 = (x - 3)(x + 5)$

Check your work by expanding the brackets and checking you get the original expression.

Your Turn

Factorise fully:

1. $x^2 + 7x + 10$
2. $x^2 + 12x + 20$
3. $x^2 + 4x - 21$
4. $x^2 - x - 6$
5. $x^2 - 13x + 30$
6. $x^2 - 10x + 25$

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Factorising: The Difference of Two Squares

If the expression consists of two square numbers separated by a minus sign then it can be factorised using the difference of two squares rule.

$a^2 - b^2 = (a + b)(a - b)$

E.g. Factorise $x^2 - 25$

$5^2 = 25$, so this becomes $(x + 5)(x - 5)$

This works even if the coefficient of x is not 1 (as long as it's square) or if both terms are algebraic.

E.g. Factorise $16x^2 - 49y^2$

$(4x)^2$ is $16x^2$ and $(7y)^2$ is $49y^2$, so this becomes $(4x + 7y)(4x - 7y)$

Your Turn

Factorise fully:

1. $x^2 - 36$

2. $a^2 - 81$

3. $4x^2 - 9$

4. $25a^2 - b^2$

5. $9x^2 - 100y^2$

6. $x^4 - y^2$

Factorising – When $a \neq 1$

This is a little more involved and, luckily, you will have a calculator that can do most of this for you. However, you still need to be able to factorise this sort of expression.

There are a number of techniques: one of which is observation; another involves reversing the process for expanding brackets.

For an expression of the form $ax^2 + bx + c$, begin by multiplying a and c to get ac. Then, find a pair of numbers whose product is ac and whose sum is b. Next, split up the middle term into two x terms with these numbers as their coefficients. Finally, factorise each *pair* of terms.

E.g. Factorise $2x^2 + 9x + 10$
$2 \times 10 = 20$

Two numbers that have a product of 20 and a sum of 9 are 4 and 5.
$2x^2 + 9x + 10 = 2x^2 + 4x + 5x + 10$

Factorise the first pair of terms and the second.
$2x^2 + 4x + 5x + 10 = 2x(x + 2) + 5(x + 2)$

Finally, fully factorise by taking out a factor of $(x + 2)$
$2x(x + 2) + 5(x + 2) = (x + 2)(2x + 5)$

Your Turn
Factorise fully:

1. $2x^2 + 11x + 12$
2. $3x^2 + 26x + 35$
3. $4x^2 + 8x - 21$
4. $3x^2 - 19x + 20$

Completing the Square

By completing the square, we can solve non-factorable quadratic equations, perform proofs and identify turning points on quadratic graphs. The completed square form for the expression $x^2 + bx + c$ is $(x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c$. In other words, we halve the coefficient of x to find the numerical part inside the brackets. Then, square this and subtract it from the bracketed expression.

E.g. Write $x^2 - 10x + 3$ in the form $(x + m)^2 + n$, where m and n are integers.

Begin by halving the coefficient of x to find the value of m. Then, square this number and subtract it.

$-10 \div 2 = -5$

$x^2 - 10x + 3 = (x - 5)^2 - 5^2 + 3$

$= (x - 5)^2 - 22$

When we have an expression in completed square form, we can find the turning point. A quadratic graph whose equation is $y = (x + m)^2 + n$ has a turning point at $(-m, n)$.

Take our previous example. The quadratic equation $y = x^2 - 10x + 3$ can be written as $y = (x - 5)^2 - 22$. This means that the quadratic graph has a turning point at (5, -22).

E.g. A curve is given by the equation $y = x^2 + 4x - 1$. Write the equation in the form $y = (x + m)^2 + n$. Hence, write down the coordinates of the turning point of this graph.

$4 \div 2 = 2$

$x^2 + 4x - 1 = (x + 2)^2 - 2^2 - 1$

$= (x + 2)^2 - 5$

The turning point has coordinates (-2, -5).

Note that, if the coefficient of x^2 is not equal to 1, you will need to factorise part of the expression first.

E.g. $3x^2 + 6x + 5 = 3(x^2 + 2x) + 5$

$= 3((x + 1)^2 - 12) + 5$

$= 3(x + 1)^2 - 3 + 5$

$= 3(x + 1)^2 + 2$

Your Turn

Write each equation in completed square form, and then find the coordinates of the turning point.

1. $y = x^2 + 8x + 23$

2. $y = x^2 - 6x + 1$

3. $y = x^2 + 4x - 6$

4. $y = x^2 + 3x + 9$

5. $y = x^2 - 5x - 8$

6. $y = 2x^2 + 12x + 7$

7. $y = 3x^2 + 12x + 2$

8. $y = 2x^2 + 6x + 23$

A-Level Preparation **Equations and Inequalities**

Simultaneous Equations

When solving simultaneous equations, we are finding the values of the given variables (usually x and y) that make both equations true. There are two techniques: elimination (this works better for linear equations) and substitution (this works better for quadratic equations but can be used in either case).

Example 1

Solve the simultaneous equations $2x - 4y = -14$ and $3x + 7y = 18$.

To use the elimination method, we need to multiply one or both equations by some constant to create a common coefficient.

Let's call $2x - 4y = -14$ equation 1 and $3x + 7y = 18$ equation 2.

Multiply equation 1 by 3 and equation 2 by 2.

$6x - 12y = -42$

$6x + 14y = 36$

To eliminate the x parts, subtract one equation from the other (if the signs of the coefficients are different then we add the equations) and then solve the resulting equation. It doesn't matter which way around we do this, as long as we are careful with the signs.

$-26y = -78$

$y = 3$

Now, substitute this value into either of the original equations and solve for x. Take equation 1:

$2x - 4 \times 3 = -14$

$2x - 12 = -14$

$2x = -2$

$x = -1$

The solution is $x = -1$ and $y = 3$.

Example 2

Solve the simultaneous equations $x^2 + y^2 = 5$ and $y - x = 3$.

To use the substitution method, rearrange the linear equation to make y the subject.

$y = 3 + x$

Now, substitute this into the quadratic equation then solve.

$x^2 + (3 + x)^2 = 5$

$x^2 + 9 + 6x + x^2 = 5$

$2x^2 + 6x + 9 = 5$

$2x^2 + 6x + 4 = 0$

We can factorise from here but it is easier to divide through by 2, then factorise.

$x^2 + 3x + 2 = 0$

$(x + 2)(x + 1) = 0$

$x = -2, x = -1$

Don't forget to substitute both of these back into the original linear equation to find the corresponding values for y.

When $x = -2$:

$y - (-2) = 3$

$y + 2 = 3$

$y = 1$

When $x = -1$:

$y - (-1) = 3$

$y + 1 = 3$

$y = 2$

The solutions are $x = -2$, $y = 1$ and $x = -1$, $y = 2$.

Your Turn

1. Solve each pair of simultaneous equations:

   a. $x + y = 14$
      $2x - y = 16$

   b. $3x + 2y = -4$
      $2x + y = -3$

c. $2x + 5y = 26$

$y = x + 1$

d. $x^2 + y^2 = 10$

$y = x + 2$

e. $2x^2 = y^2 - 8$

$y - x = 2$

BEYOND MATHS

A-Level Preparation **Equations and Inequalities**

The Quadratic Formula

For quadratic equations that cannot be factorised, the quadratic formula can help. For an equation of the form $ax^2 + bx + c = 0$, the solutions are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that, since the equation includes a square root, there won't always be real solutions to a quadratic equation. The part inside the square root sign is called the discriminant and is referred to as the delta symbol Δ. It follows that if this is less than zero then there are no real solutions to the equation. If it is equal to zero, there is one solution, and if it is greater than zero, there are two.

Example 1

Solve the equation $3x^2 + 5x - 4 = 0$, giving your answers correct to 3 significant figures.

$a = 3$, $b = 5$ and $c = -4$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 3 \times -4}}{2 \times 3}$$

$x = -2.26$ or $x = 0.591$

Your Turn

1. Solve $2x^2 + 5x + 1 = 0$, giving your answers correct to 3 significant figures.

2. Solve $5x^2 + 2x = 19$, giving your answers correct to 3 significant figures.

BEYOND MATHS

3. Explain why the graph of the equation $y = x^2 + 4x + 9$ does not intersect the x-axis.
   Use the discriminant $\Delta = b^2 - 4ac$

A-Level Preparation **Simplifying Surds**

When you find the square root of an integer, your answer will be one of two types of number. If you find the root of a square number, your result will be an integer: $\sqrt{4} = 2$; if you find the square root of any other integer, your answer will be irrational: $\sqrt{2} = 1.414213562373095...$

A rational number is any number that can be represented as a fraction of two integers, e.g. $\frac{1}{3}$ (even though it recurs) or $\frac{3}{1}$ (the integer 3), while an irrational number is any number that is not rational.

An irrational number will never end and never repeat so if a square root results in an irrational number, you either have to round your answer to write it, or leave it as a root.

If you round it, you have lost some of the accuracy of the answer. A surd is therefore an irrational number that has been left in the form of a root to represent its exact value.

For example, we want to find the value of $5x^2 + 3$, where $x = \sqrt{6}$.

We have two options: 1. Use $x = \sqrt{6}$;

2. Use $x = 2.4$ ($\sqrt{6}$ rounded to 1d.p.)

1. $5(\sqrt{6})^2 + 3 = 5 \times 6 + 3 = 33$
2. $5 \times 2.4^2 + 3 = 5 \times 5.76 + 3 = 31.8$

Even with an example with small numbers, you can see how different the rounded answer is. In any situation dealing with roots, it is therefore important that you can operate with numbers in surd form.

To do this, you need to be able to simplify surds, starting with multiplying and dividing. Here are some examples:

E.g. a. $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3}$

$= \sqrt{6}$

b. $2\sqrt{5} \times 4\sqrt{2} = 2 \times 4 \times \sqrt{5 \times 2}$

$= 8\sqrt{10}$

c. $\sqrt{10} \div \sqrt{5} = \sqrt{10 \div 5}$

$= \sqrt{2}$

d. $\sqrt{10} \div \sqrt{3} = \sqrt{\frac{10}{3}}$

When multiplying surds, you simply multiply the base of each root.

If there are coefficients outside the root, multiply them separately.

You can divide the bases if the result is an integer. If the result is not an integer, you will normally write the surds as a fraction.

BEYOND MATHS

You try some:

1. a. $\sqrt{5} \times \sqrt{7}$

d. $18\sqrt{20} \div 6\sqrt{5}$

g. $(5\sqrt{3})^2$

b. $3\sqrt{2} \times 4\sqrt{5}$

e. $5\sqrt{2} \times 3\sqrt{8}$

h. $(2\sqrt{5})^3$

c. $\sqrt{15} \div \sqrt{3}$

f. $2\sqrt{3} \times 5$

2. A right-angled triangle has a height of $6\sqrt{5}$cm and a base of $7\sqrt{3}$cm. Find its area.

Addition and subtraction of surds can be more complex. In the same way you cannot add or subtract fractions with different denominators, you cannot simply add or subtract surds with different bases:

E.g. a. $3\sqrt{5} + 8\sqrt{5} = 11\sqrt{5}$

b. $2\sqrt{5} + 8\sqrt{3} \neq 10\sqrt{8}$

In some cases, such as example b above, the addition is not possible and you would leave your answer as $2\sqrt{5} + 8\sqrt{3}$. In other cases, you can simplify one surd so it has the same base as the other:

E.g. $\sqrt{2} + \sqrt{8}$

Initially, this might look impossible. However, 8 has a square factor (4 × 2). This means:

$\sqrt{8}$

$= \sqrt{4 \times 2}$

$= \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

The key to simplification is to find a square factor. Since we know $\sqrt{4} = 2$, we can write $\sqrt{8}$ as $2\sqrt{2}$, giving:

$$\sqrt{2} + \sqrt{8}$$
$$= \sqrt{2} + 2\sqrt{2}$$
$$= 3\sqrt{2}$$

Here's another example:

$$3\sqrt{3} + 2\sqrt{12}$$
$$= 3\sqrt{3} + 2\sqrt{3 \times 4}$$ Look for a square factor.
$$= 3\sqrt{3} + 2(\sqrt{4}\,\sqrt{3})$$
$$= 3\sqrt{3} + 2(2\sqrt{3})$$
$$= 3\sqrt{3} + 4\sqrt{3}$$
$$= 7\sqrt{3}$$

Your turn:

1. Simplify these surds (remember: the key is to find a square factor).

   a. $\sqrt{20}$ ____________
   b. $\sqrt{48}$ ____________
   c. $\sqrt{75}$ ____________
   d. $5\sqrt{8}$ ____________

2. Answer the following, giving your answers in the form $a\sqrt{b}$

   a. $\sqrt{2} + \sqrt{18}$

   ____

   ____

   b. $\sqrt{50} - \sqrt{200}$

   ____

   ____

   c. $4\sqrt{80} + 3\sqrt{45}$

   ____

   ____

   d. $2\sqrt{50} + 5\sqrt{32}$

   ____

   ____

3. A rectangle has a width of $6\sqrt{75}$m and a height of $2\sqrt{12}$m. What is its perimeter?

Challenge

A right-angled triangle has a base of $2\sqrt{18}$cm and a height of $2\sqrt{32}$cm. Find the perimeter of the triangle.

A-Level Preparation **Rationalising the Denominator**

A surd is a way of expressing an irrational root without losing accuracy. When a surd is on the denominator of a fraction, this fraction can be simplified by replacing that surd with an integer. This is called “rationalising” the denominator.

Consider this fraction:

$$\frac{1}{\sqrt{2}}$$

As 2 is not a square number, $\sqrt{2}$ is irrational and we want to remove it from the denominator. We can do this by multiplying our fraction by $\frac{\sqrt{2}}{\sqrt{2}}$.

As $\frac{\sqrt{2}}{\sqrt{2}}$ cancels to 1, this multiplication does not change the value of the fraction.

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$ Multiply the numerators, multiply the denominators.

$$= \frac{\sqrt{2}}{2}$$ $(\sqrt{2} \times \sqrt{2} = 2)$

There is now a surd in the numerator but the denominator is a rational number, 2. Here are some more examples:

E.g. a. Rationalise the denominator of $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

b. Rationalise the denominator of $\frac{7}{\sqrt{6}}$

$$\frac{7}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{7\sqrt{6}}{6}$$

Your Turn:

1. Rationalise the denominator of each fraction.

a. $\frac{5}{\sqrt{2}}$ ____________

c. $\frac{\sqrt{2}}{\sqrt{3}}$ ____________

e. $\frac{2 - \sqrt{3}}{\sqrt{3}}$ ____________

b. $\frac{4}{\sqrt{3}}$ ____________

d. $\frac{3}{2\sqrt{5}}$ ____________

BEYOND MATHS

2. Rationalise the denominator of $\frac{\sqrt{5}}{\sqrt{80}}$. Give your answer as a fraction in its simplest form.

3. What is $\frac{2\sqrt{2}}{\sqrt{6}} + \frac{1}{\sqrt{3}}$? Give your answer in its simplest terms.

A similar method can be used to rationalise fractions with more complicated denominators, such as:

$$\frac{2}{\sqrt{3} + 4}$$

In this case, multiplying numerator and denominator by $\sqrt{3}$ will give us:

$$\frac{2\sqrt{3}}{3 + 4\sqrt{3}}$$

This has not rationalised the denominator so we need to multiply by something different.

Consider what happens when you expand a pair of brackets in the form $(a + b)(a - b)$:

$(a + b)(a - b)$

$= a^2 + ab - ab - b^2$

$= a^2 - b^2$

In this case, all we are left with is the difference of two squares. If a or b were surds, they would now be rational.

We can apply this technique to rationalising denominators.

$\frac{2}{\sqrt{3}+4} \times \frac{\sqrt{3}-4}{\sqrt{3}-4}$ Multiply numerator and denominator by $\sqrt{3} - 4$. Notice we have changed the sign on the denominator (this is called the conjugate).

$= \frac{2\sqrt{3}-8}{(\sqrt{3}+4)(\sqrt{3}-4)}$ Start by expanding the brackets in the numerator

$= \frac{2\sqrt{3}-8}{\sqrt{3}\sqrt{3}+4\sqrt{3}-4\sqrt{3}-16}$ Then, expand the brackets in the denominator

$= \frac{2\sqrt{3}-8}{-13}$ or $\frac{8-2\sqrt{3}}{13}$ Finally, simplify the denominator. Remember: $\sqrt{3} \times \sqrt{3} = 3$

The surd is now rationalised.

Your Turn:

1. a. $\frac{5}{\sqrt{2}+7}$ b. $\frac{1}{\sqrt{5}-3}$ c. $\frac{1+\sqrt{2}}{\sqrt{3}+2}$

Challenge

Amy is laying tiles in her rectangular bathroom. By the time she has finished, she has used $8m^2$ worth of tiles. She knows the length of one side of the room is $(\sqrt{5} + 2)$m but, unfortunately, she has lost her tape measure. Amy still needs to work out the perimeter of the room. Calculate the perimeter of the room, giving your answer in its simplest form.

A Level Preparation **Graphs**

Equations of Straight-Line Graphs

The general equation of a straight line is $y = mx + c$, where m represents the gradient (steepness) of the line and c is the value of the y-intercept. You might need to rearrange an equation to be able to calculate one of these.

Example 1

Find the gradient and the coordinates of the y-intercept of the line with equation $3x + 2y = 6$.

Begin by rearranging to make y the subject.

$2y = 6 - 3x$

$y = 3 - \frac{3}{2}x$

The gradient is $-\frac{3}{2}$ and the y-intercept is 3, so its coordinates are (0, 3).

You can find the gradient of a straight line given any two points on that line. If those two points are (x_1, y_1) and (x_2, y_2), the formula for the gradient of a straight line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Parallel and Perpendicular Lines

Two lines are parallel if their gradients are equal, and they are perpendicular if the product of their gradients is -1. In other words, if m_1 and m_2 are the gradients of two lines, these lines are perpendicular if $m_1 \times m_2 = -1$.

Example 2

Prove that the line passing through the points (4, 1) and (2, 6) is perpendicular to the line whose equation is $5y = 2x + 3$.

The gradient of the first line is $m = \frac{6 - 1}{2 - 4} = -\frac{5}{2}$

The equation of the second line can be written as $y = \frac{2}{5}x + \frac{3}{5}$, so its gradient is $\frac{2}{5}$

$-\frac{5}{2} \times \frac{2}{5} = -1$ therefore the lines are perpendicular.

Note that it's sensible to leave any non-integer values in fractional form. Fractions are *generally* easier to work with and if the equivalent decimal is non-terminating then they also provide an exact solution.

BEYOND MATHS

If you know the gradient of a line and the coordinates of at least one point, you can find the equation. Substitute everything you know into the equation of the line and then solve for c.

Example 3

Find the equation of the line which is parallel to the line with equation $y = 4x + 2$ and which passes through (3, 7).

The gradient of the line given is 4 and the lines are parallel, so the gradient of our line is also 4.

Substitute this value into the equation $y = mx + c$ to get $y = 4x + c$.

We know the line passes through (3, 7), so substitute $x = 3$ and $y = 7$ into this equation and solve for c.

$7 = 4 \times 3 + c$

$7 = 12 + c$

$c = 7 - 12 = -5$

The equation is $y = 4x - 5$

Your Turn

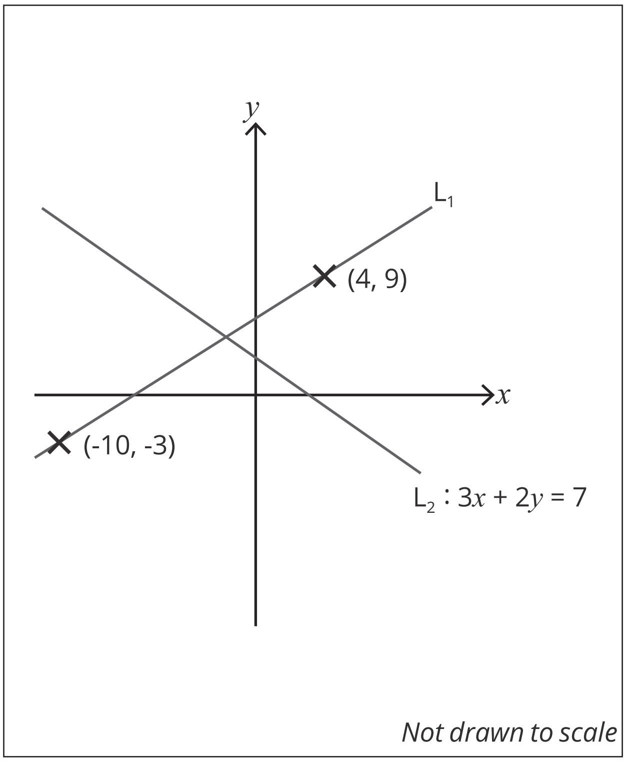
1. Complete the table:

Equation	Gradient	y-intercept
$y = 3x + 7$		
$y = 2 - x$		
$2y = x + 5$		
$3y + 2x = -1$		
	-2	0
	$\frac{3}{4}$	-1

2. Find the equation of the line passing through the points with coordinates (2, 3) and (4, 1).

3. A line whose gradient is $\frac{1}{3}$ passes through the point (-6, 9). Work out the equation of this line, giving your answer in the form $ay + bx = c$, where a, b and c are integers.

4. The diagram shows two straight lines. Are the lines perpendicular? Justify your answer.

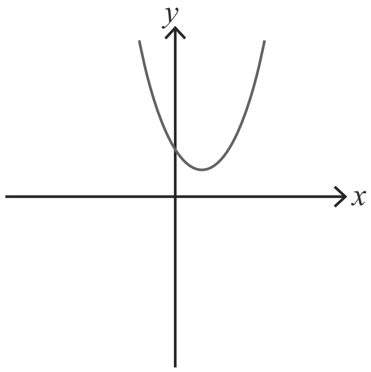


5. Does the line with equation $2x + 5y = -1$ pass through the point with coordinates (2, -1)?

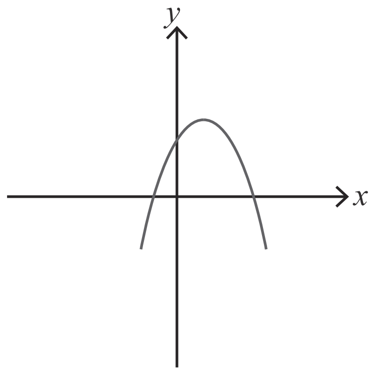
Quadratic Graphs

All quadratic graphs are parabolas: a symmetrical curve. The orientation of this parabola depends on whether the coefficient of x^2 is positive or negative.

If the coefficient of x^2 is positive, the graphs are u-shaped:



If the coefficient of x^2 is negative, the graphs are n-shaped:



We can use our skills for working with quadratic equations to find other key features of the graph:

1) The y-intercept is found by setting $x = 0$ and solving the given equation for y.
2) The x-intercept is found by setting $y = 0$ and solving the given equation for x.
3) The turning point (which will be the minimum or maximum of a quadratic function) can be found by completing the square. This can also be found by finding the average of the x-intercept values but completing the square can be more efficient. A graph with an equation of the form $y = a(x + b)^2 + c$ has a turning point at $(-b, c)$.

For example,

Sketch the graph of $y = x^2 – 7x + 10$, clearly indicating any points of intersection with the axes and the location of the turning point of the curve.

The coefficient of x^2 is 1. This is positive, so our graph will be u-shaped.

The y-intercept is found by substituting $x = 0$ into the equation.

$y = 0^2 - 7 \times 0 + 10$

$y = 10$

The x-intercept is found by substituting $y = 0$ into the equation then solving the resulting equation.

$0 = x^2 - 7x + 10$

$0 = (x - 2)(x - 5)$

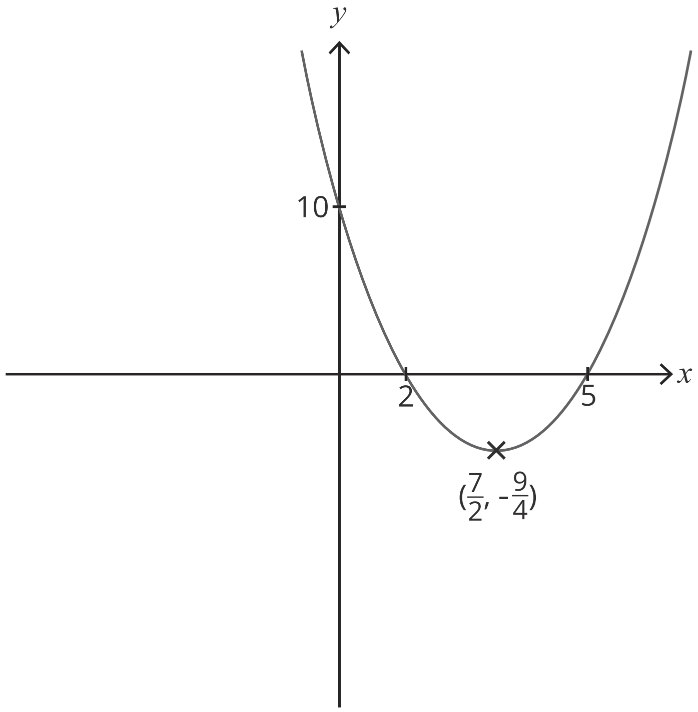
$x = 2, x = 5$

The turning point is found by completing the square.

$$\begin{aligned} x^2 - 7x + 10 &= (x - \tfrac{7}{2})^2 - (\tfrac{7}{2})^2 + 10 \\ &= (x - \tfrac{7}{2})^2 - \tfrac{9}{4} \end{aligned}$$

The turning point has coordinates $(\frac{7}{2}, -\frac{9}{4})$.

When sketching a graph, it does not need to be to scale but should be the right shape and roughly in proportion as shown:



BEYOND MATHS

Your Turn

1. Consider the curve with equation $y = x^2 + 4x - 5$.

   a. Find the coordinates of the point where this curve intersects the y-axis.

   b. Find the coordinates of the points where this curve intersects the x-axis.

   c. Hence, sketch the graph of $y = x^2 + 4x - 5$, clearly indicating any points of intersection with the axes.

2. Consider the curve with equation $y = x^2 + 8x - 1$.

   a. Find the coordinates of the turning point of this curve.

   b. State whether the turning point is a maximum or minimum. Justify your answer.

3. Sketch the graph of $y = x^2 + 4x - 21$, clearly indicating any points of intersection with the axes and the location of the turning point of the curve.

4. Sketch the graph of $y = -x^2 + 7x$, clearly indicating any points of intersection with the axes.

5. Sketch the graph of $y = 2x^2 + 17x + 8$, clearly indicating any points of intersection with the axes.

A-Level Preparation **Right-Angled Trigonometry**

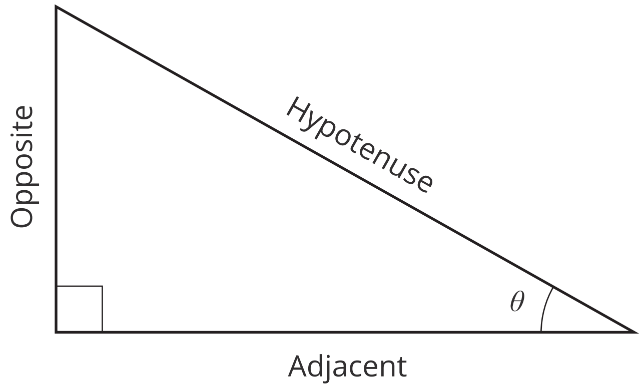
Right-angled trigonometry allows you to find a missing angle or side in a right-angled triangle when given two sides, or an angle and a side. Trigonometry uses the three trigonometric functions: sine, cosine and tangent, each of which can be expressed as a ratio of the length of two sides:

$$\sin\theta = \frac{O}{H} \qquad \cos\theta = \frac{A}{H} \qquad \tan\theta = \frac{O}{A}$$

Here, θ is the measure of the angle, O is the length of the opposite side, A is the length of the adjacent side and H is the length of hypotenuse.

The hypotenuse is always the side opposite the right angle and is always the longest side (this can be a good tool to check your answers make sense).

The opposite and adjacent sides are not fixed; they depend on the angle you are interested in. The opposite side is farthest from the angle (but not the hypotenuse). The adjacent side is next to the angle (but, again, not the hypotenuse).



Mnemonics

One of the challenges of solving a trigonometric problem is remembering the ratios. Some people use a mnemonic such as *SOH-CAH-TOA*. When written in the following format, this also lets you easily rearrange the formulae:

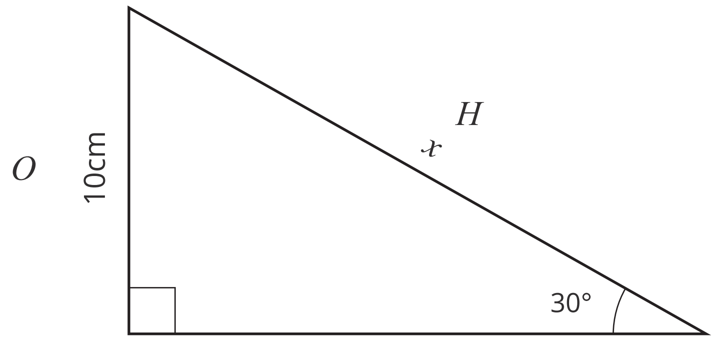
$$\frac{O}{S \times H} \qquad \frac{A}{C \times H} \qquad \frac{O}{T \times A}$$

Finding a Missing Side

Your first step in solving a right-angled trigonometric problem will be to label the sides of the triangle.

Your second step will be to choose the relevant ratio. In a typical problem, you will have a missing value and at least two known values. Consider the example below:

Find the length of side x.



Start by labelling the sides. In this case, we have the opposite (O) and hypotenuse (H). We know the angle and the opposite side but we are interested in the hypotenuse. Therefore, we need to choose the trigonometric ratio that includes O and H:

$$\sin\theta = \frac{O}{H}$$

We are looking for H, so we must rearrange our formula:

$$H = \frac{O}{\sin\theta}$$

Then, substitute our values and calculate our answer:

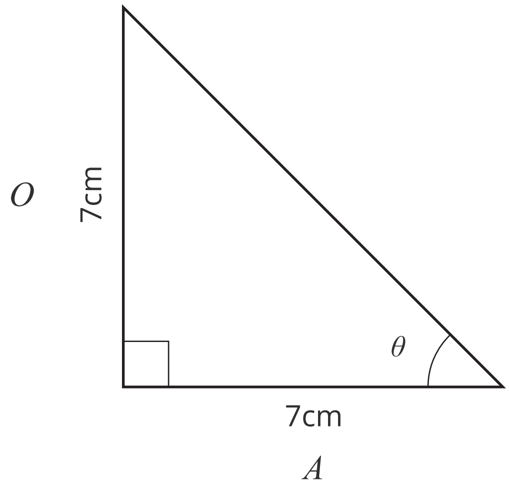
$$x = \frac{10}{\sin(30°)}$$

$$x = 20\text{cm}$$

We could also choose to rearrange the formula after substituting the values. This is entirely up to you, and won't affect the answer.

Finding a Missing Angle

In a similar way, you can find a missing angle when given two sides:



As before, start by labelling the sides. In this case, we have the opposite (O) and adjacent (A) sides. We need to use these to find the value of θ. We need to choose the trigonometric ratio that includes O and A:

$$\tan\theta = \frac{O}{A}$$

As before, we substitute our values:

$$\tan\theta = \frac{7}{7}$$

$$\tan\theta = 1$$

We have one more step. We are interested in the size of the angle θ but we still only know $\tan\theta$. To find θ, we need to carry out the inverse tan function. This is normally written as $\tan^{-1}$ or, sometimes, as arctan:

$$\tan\theta = 1$$

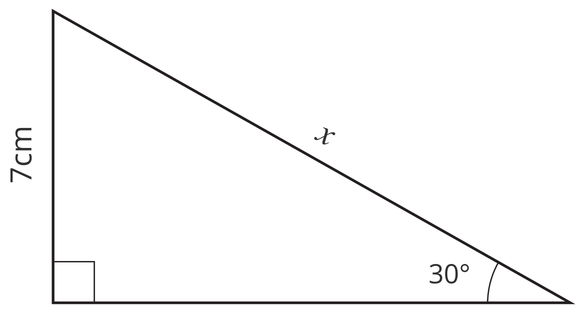
$$\theta = \tan^{-1}(1)$$

$$\theta = 45°$$

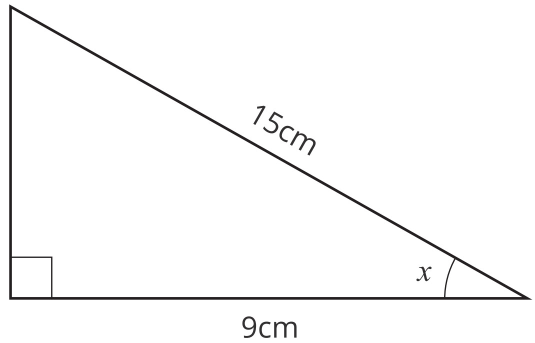
Your Turn:

1. In each question, find the value of x. Give your answers to 3s.f.

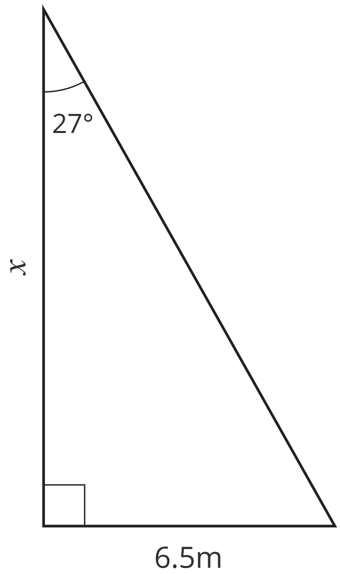
a.



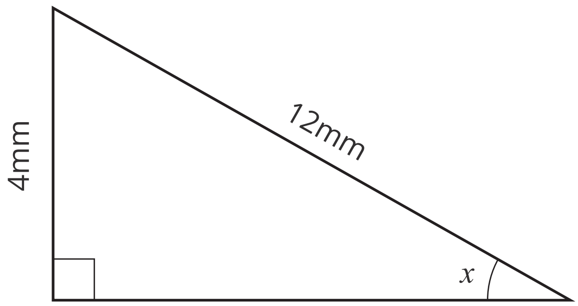
b.



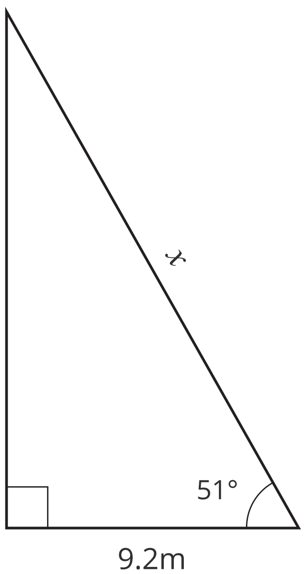
c.



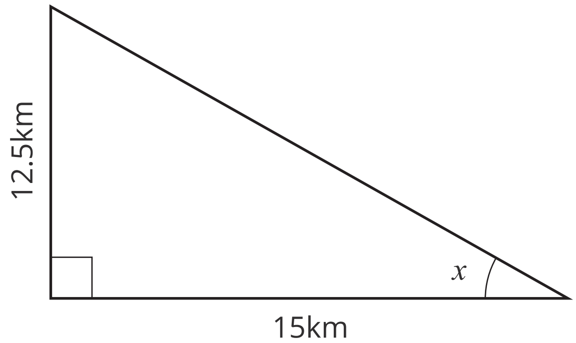
d.



e.



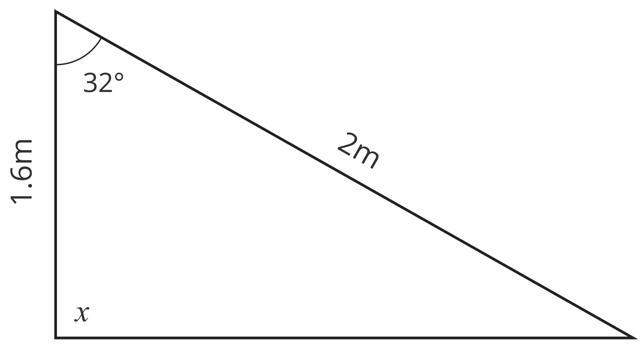
f.



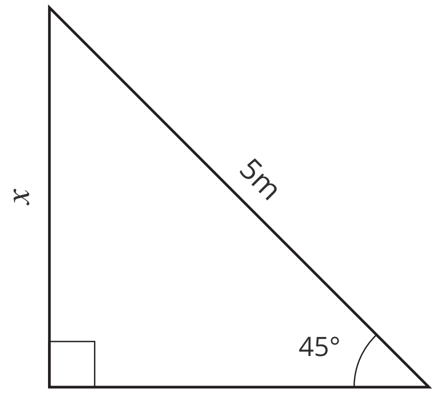
2. Zara is an engineer. She is building a wall, which needs to be supported as it is built. The ground is perfectly horizontal, the wall is perfectly vertical and the support is a straight steel beam. For the wall to be safe, the angle between the beam and the ground must be less than 55°.

   If the support beam is 12m long and the end of the support is 3.5m away from the base of the wall, is the wall safely supported?

3. Look at the diagram below. Is x a right angle? You must justify your answer.



4. Find the length of side x, giving your answer as a surd in its simplest form.



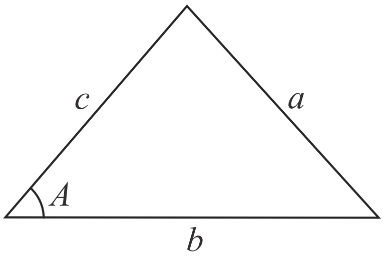
5. A mnemonic that can help to remember the trigonometric ratios is: **S**ydney **o**pera **h**ouse **c**an **a**lways **h**old **t**housands **o**f **A**ustralians. Try to make up your own mnemonic.

A Level Preparation **Sine and Cosine Rules**

The sine and cosine rules use trigonometric functions to find the size of missing angles or sides in any triangle. Unlike right-angled trigonometry, you do not need to have a right-angled triangle to apply them.

The Cosine Rule

For any triangle:



You may notice the similarity to Pythagoras' theorem. The cosine rule essentially uses the $-2bc\cos A$ term to apply Pythagoras' theorem to non-right-angled triangles. In a right-angled triangle, A is 90°, cos(90°) is 0 and so $-2bc\cos A$ is zero, leaving $a^2 = b^2 + c^2$.

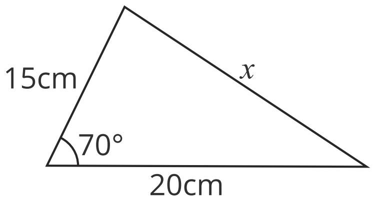
The cosine rule is

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

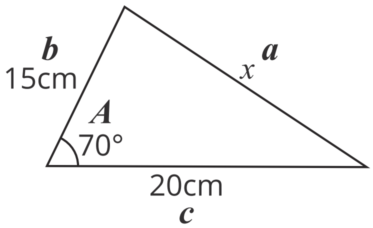
When using the cosine rule to find a missing side, you need the length of both adjacent sides and the angle opposite the side you are interested in.

Example 1

Find the length of side x. Give your answer correct to 3s.f.



Your first step is to label the sides and angles. Label the side you are interested in, a, and the angle opposite as A. Label the other two sides b and c.



Substitute your values into the formula and solve for x:

$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 15^2 + 20^2 - 2 \times 15 \times 20 \times \cos(70°)$

$x^2 = 419.787...$

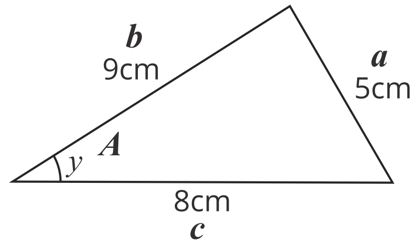
$x = 20.5$cm

You can also use the cosine rule to find the size of an angle by rearranging the formula (or you may choose to learn the rearranged version of the formula by heart). To do this, you need to know the length of the side opposite the angle.

BEYOND MATHS

Example 2

Find the size of angle y. Give your answer correct to 1d.p.



In this case, the sides are already labelled. Remember: side a must be opposite angle A but sides b and c can be either way round. We now need to rearrange our formula:

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 - b^2 - c^2 = -2bc\cos A$ Subtract b^2 and c^2 from both sides.

$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$ Divide both sides by $-2bc$.

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ Multiply top and bottom by $\frac{-1}{-1}$ to tidy up the negatives.

Now, you can substitute your values and solve for y:

$\cos y = \frac{9^2 + 8^2 - 5^2}{2 \times 9 \times 8}$

$\cos y = \frac{120}{144}$

$y = \cos^{-1}(\frac{120}{144})$

$y = 33.6°$ (to 1d.p.)

Sine Rule

In problems where you don't have enough information to use the cosine rule, you can use the sine rule instead. The sine rule says that within a triangle, the ratio of the length of each side to the sine of the opposite angle is equal:

$$\boxed{\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}}$$

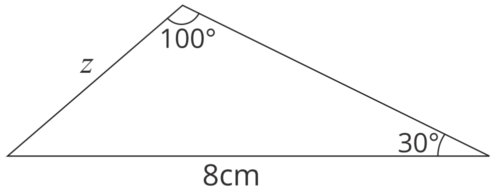
Or, if you take the reciprocal of each fraction:

$$\boxed{\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}}$$

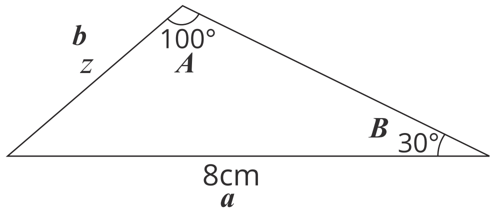
Where A is the angle opposite side a, B is the angle opposite side b and C is the angle opposite side c. We can then choose the sides we need for a question.

Example 3

Find the length of side z. Give your answer correct to 2d.p.



Start by labelling your sides:



We haven't labelled angle C or side c in this question as we don't need to use them. In the same way, we will remove them from the formula, leaving:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

Substitute your values and solve for z:

$$\frac{8}{\sin(100°)} = \frac{z}{\sin(30°)}$$

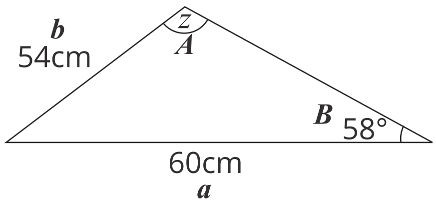
$$z = \frac{8 \times \sin(30°)}{\sin(100°)}$$

z = 4.06cm (to 2d.p.)

We can also use the sine rule to find a missing angle:

Example 4

Find the size of angle z. Give your answer correct to 3.s.f.



The sides are already labelled. This time, we are looking for an angle, so we use the formula:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

Substitute your values and solve as before:

$$\frac{\sin(z)}{60} = \frac{\sin(58°)}{54}$$

$$\sin(z) = \frac{6 \times \sin(58°)}{54}$$

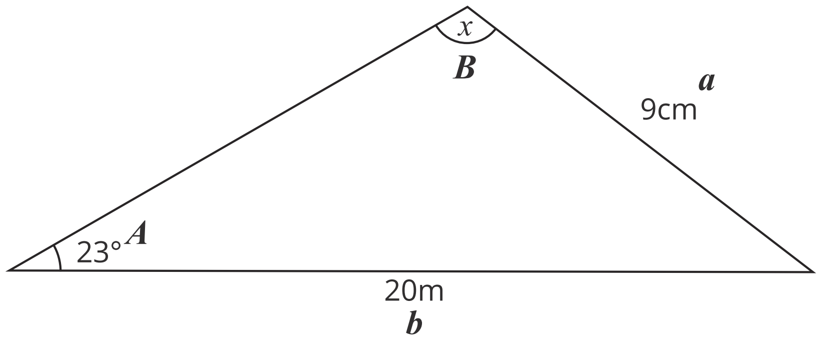
$z = \sin^{-1}(0.942...)$

z = 70.4° (to 3s.f.)

There are some situations in which you will need to be more careful when deciding how to apply the sine rule. Consider the following example:

Example 5

Find the size of obtuse angle x. Give your answer correct to 1d.p.



$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

$$\frac{\sin(23°)}{9} = \frac{\sin(x)}{20}$$

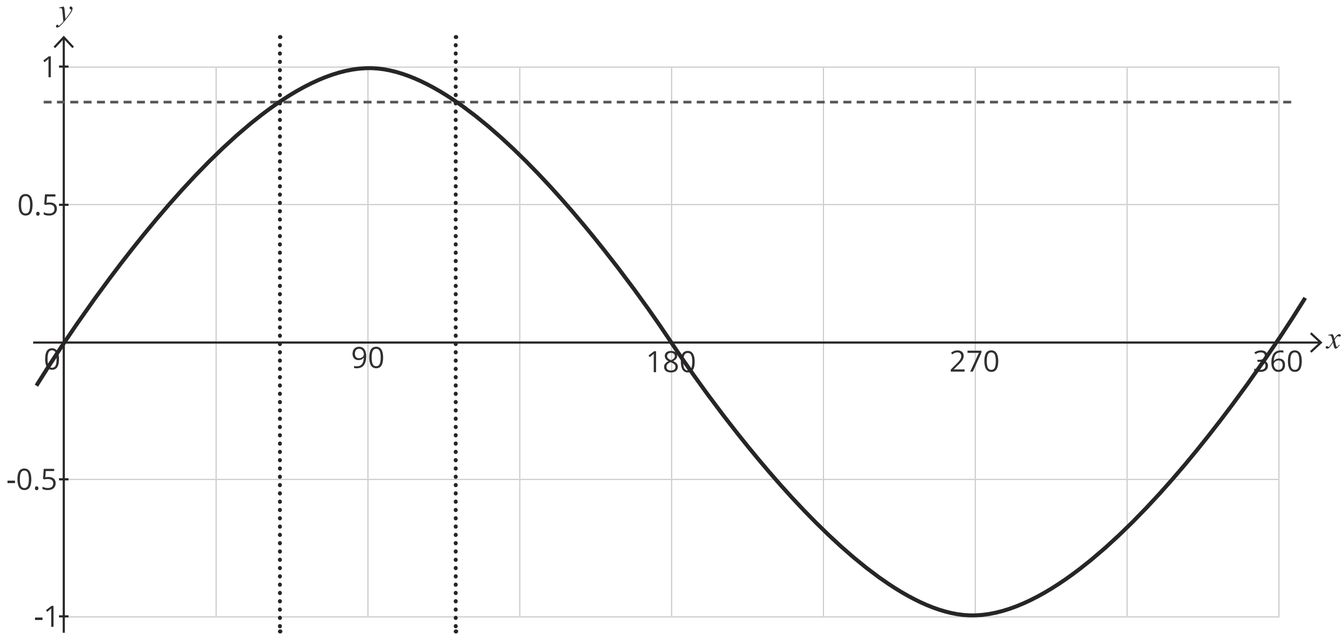
$$\sin(x) = \frac{20 \times \sin(23°)}{9}$$

$x = \sin^{-1}(0.868...)$

$x = 60.260...°$

At this point, you have to be careful. In non-right-angled triangles, the size of an angle can exceed 90°. While our calculations are correct, the answer is an acute angle, and the question states that angle x is obtuse.

Consider the graph of $y = \sin(x)$:



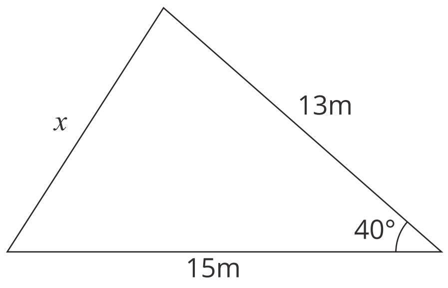
An angle in a triangle must be less than 180°. Within this range, there are two possible values for $\sin^{-1}(0.868)$ and we want to make sure we choose the one that corresponds to our angle. **This will not always be the answer given by your calculator**. In this case:

$x = 180 - 60.260...$

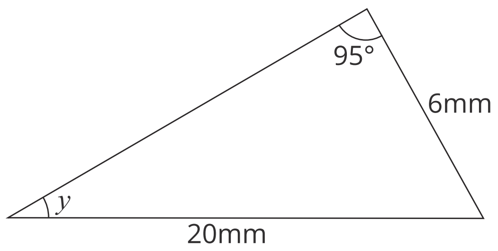
$x = 119.7°$

Your Turn

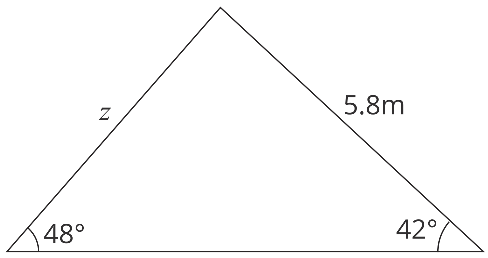
1. Find the length of side x, giving your answer correct to 1d.p.



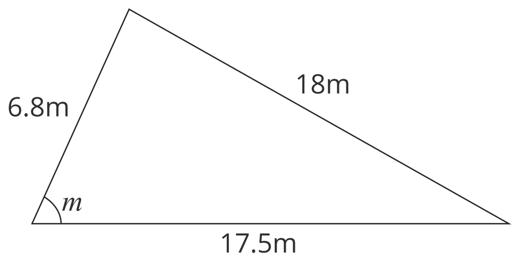
2. Find the size of angle y, giving your answer correct to 3s.f.



3. Find the length of side z, giving your answer correct to 1d.p.



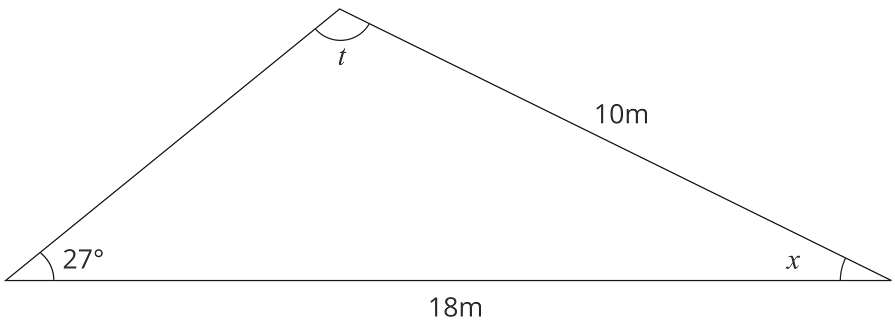
4. Find the size of angle m. Give your answer correct to 3s.f.



BEYOND MATHS

5. A plane takes off from Airport A, flies 200km due north, then turns on a bearing of 030° and flies a further 350km before landing at Airport B. How far is airport A from airport B in a straight line? Give your answer correct to the nearest kilometre.

6. Given that angle t is obtuse, find the size of angle x, giving your answer correct to 1d.p.



7. A triangle XYZ is divided by a line YW. Side YZ is 8cm, angle XYW is 30° and angle YWZ is 65°. Find the size of angle WZY, giving your answer correct to 3s.f.